STEADY-STATE DISTRIBUTION OF CHARGED PARTICLES IN THE DIFFUSION APPROXIMATION

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The steady-state distribution of charged particles in a weakly ionized plasma is examined for the case in which volume ionization, recombination, and diffusion in a space-charge field occur. A joint solution is obtained for the equation for charged-particle balance and the Poisson equations for the case of planar and cylindrical plasma configurations satisfying the Schottky condition at the boundaries of the region. A solution is also found for the case in which the ionization is localized in a spherically symmetric volume and in which the Schottky condition is satisfied at infinity. The condition for the existence of a steady-state solution is given and analyzed.

The steady-state distribution of charged particles in a weakly ionized plasma can be obtained for the balance equations for the number of particles during volume ionization, recombination, and diffusion in a space-charge field. The basic removal mechanism in a low-density plasma at sufficiently low particle densities is the independent diffusion of electrons and ions toward the periphery, with subsequent recombination at the wall (free diffusion). It is usually assumed that the ionization frequency per unit volume is proportional to the electron concentration. If the mean free path is small in comparison with the typical dimension of the region under consideration, the problem reduces to the linear diffusion equation with homogeneous boundary conditions.

For a dense plasma at high particle concentrations one must take into account several additional effects, of which the most important are the effects of the electric field of the space charge and volume recombination; accordingly, the problem becomes highly nonlinear. The divergence of the space-charge field is proportional to the difference between the ion and the electron densities, and the simplest account of the effect of the space-charge field is based on the assumption that the ratio $C = N/N_+$ of the electron and ion densities is constant (C is some constant) at all points in the region. If $C = 1$, i. e., if the densities are equal, we find the familiar ambipolar-diffusion conditions.

If we denote the ion and electron diffusion coefficients by D_+, D, respectively, we will have free diffusion when $Q = D_+/D$. The nonlinear problem for the transition from free diffusion to ambipolar diffusion was analyzed by Allis and Rose [1] without an account of volume recombination. With regard to volume recombination, we note that in [2, 3] the diffusion was assumed ambipolar everywhere in the volume; this is an important simplification of the problem. In a region in which there are N_+, N positive and negative particles per cubic centimeter, the recombination of these particles can be described by $\alpha N_+ N$, where α is the radiation-recombination coefficient. During ambipolar diffusion, the recombination term simplifies, becoming αN^2. The recombination coefficient depends on the type of particles participating in the recombination. However, by assigning a slightly different meaning to the coefficient α, we can easily take into account volume ionization, stepped ionization, and several other processes occurring in a weakly ionized plasma.

1. We make use of the continuity equation for the number of particles per unit volume and the Poisson equation. Neglecting neutral-gas transfer and assuming the degree of ionization to be small ($N/N_n \ll 1$), we have, for electrons and ions,

$$\nabla (N\mathbf{v}) - \alpha N N_+ + ZN = 0 \quad \left(\mathbf{v} = -D\,\frac{\nabla N}{N} - b\mathbf{E}\right), \tag{1.1}$$

$$-\nabla (N_+\mathbf{v}_+) - \alpha N N_+ + ZN = 0 \quad \left(\mathbf{v}_+ = -D_+\,\frac{\nabla N_+}{N_+} + b_+\mathbf{E}\right), \tag{1.2}$$

$$\nabla \mathbf{E} = 4\pi e\,(N_+ - N)\,. \tag{1.3}$$

Here N and N_+, the electron and ion densities, respectively, are functions of the coordinates; E is the electric field of the space charge; and v and v_+ are the average drift velocities of electrons and ions. The volume-recombination coefficient α and the frequency Z for electron-impact ionization, like the diffusion coefficients D_+ and D and mobilities b and b_+ corresponding to electrons and ions, are assumed independent of the coordinates. This assumption corresponds to the assumption of constant electron and ion temperatures T and T_+ throughout the volume. We supplement system (1.1)–(1.3) by the Schottky boundary conditions:

$$N|_{\Sigma} = N_+|_{\Sigma} = 0 \, . \tag{1.4}$$

We note that analogous equations were discussed in [4] in an analysis of the plasma-perturbation region near an electrode.

The problem will be treated below for the cases of planar, cylindrical, and spherical symmetry, so the boundaries of the plasma region will be either two infinite parallel planes with coordinates L/2 − L/2 or a cylindrical or spherical surface of radius R. Here all the variables in (1.1)–(1.3) depend only on a single coordinate x_i ($i = 1, 2, 3$), ($x_1 = x$, $x_2 = \rho$, $x_3 = r$). Simple physical considerations based on the symmetry of the problem and the requirement that the solution be bounded lead to the conditions

$$\nabla N = \nabla N_+ = 0, \; \nabla^2 N, \; \nabla^2 N_+ < 0, \; E = 0 \text{ for } x_i = 0 \, . \tag{1.5}$$

For sufficiently low densities N and N_+ all the nonlinear terms in (1.1) and (1.2) can be neglected, and we have, instead of (1.1)–(1.3),

$$D\nabla^2 N + ZN = 0, \; D_+\nabla^2 N_+ + ZN = 0 \, , \tag{1.6}$$

which are linear diffusion equations describing, with conditions (1.4) and (1.5), the well-known free-diffusion regime in which volume recombination and the effect of the space-charge field can be neglected. The solution of Eq. (1.6) can be written

$$N = N_0 G_i(\xi_i), \; \xi_i = x_i / \Lambda_i, \; \Lambda_i = (D / Z)^{-1/2} \, ,$$

where Λ_i is the diffusion length, and i = 1, 2, 3 for planar, cylindrical, and spherical symmetry, respectively.

Equations (1.6) show that the ion concentration is related to the electron concentration by $N_+ = D/D_+ N$.

For i = 1 (planar symmetry), we have

$$\Delta_1 = L / \pi, \; G_1 = \cos x / \Lambda_1 \, ; \tag{1.7}$$

for i = 2 (cylindrical symmetry), we have

$$\Lambda_2 = R/\mu, \; G_2 = J_0(\rho / \Lambda_2) \, , \tag{1.8}$$

where μ = 2.405 is the first root of the zero-order Bessel function; for i = 3 (spherical symmetry), we have

$$\Lambda_3 = R/\pi, \; G_3 = \Phi(r/\Lambda_3) = \sqrt{\pi\Lambda_3/2r} \, J_{1/2}(r/\Lambda_3) \, , \tag{1.9}$$

where Φ is the spherical Bessel function of zeroth order.

2. We write Eqs. (1.1)–(1.3) in dimensionless form:

$$\frac{n}{\xi_i^{i-1}} \frac{d}{d\xi_i}(\xi_i^{i-1}\varepsilon) + \frac{a_0^{(i)}}{\xi_i^{i-1}} \frac{d}{d\xi_i}\left(\xi_i^{i-1}\frac{dn}{d\xi_i}\right) + \varepsilon\frac{dn}{p\xi_i} - a_1 n n_+ + a_2 n = 0 \, , \tag{2.1}$$

$$\frac{n_+}{\xi_i^{i-1}} \frac{d}{d\xi_i}(\xi_i^{i-1}\varepsilon) - \frac{\mu}{\lambda}\frac{a_0^{(i)}}{\xi_i^{i-1}} \frac{d}{d\xi_i}\left(\xi_i^{i-1}\frac{dn_+}{d\xi_i}\right) + \varepsilon\frac{dn}{d\xi_i} + \mu a_1 n n_+ - \mu a_2 n = 0 \, , \tag{2.2}$$

$$\frac{1}{\xi_i^{i-1}} \frac{d}{d\xi_i}(\xi_i^{i-1}\varepsilon) = 4\pi(n_+ - n) \, , \tag{2.3}$$

$$a_0^{(i)} = \frac{D}{eb\Lambda_i}, \quad a_1 = \frac{\alpha}{eb}, \quad a_2 = \frac{Z}{ebN_n}, \quad \mu = \frac{b}{b_+}, \quad \lambda = \frac{D}{D_+},$$

$$n = \frac{N}{N_n}, \quad n_+ = \frac{N_+}{N_n}, \quad \xi_i = \frac{x_i}{\Lambda_i}, \quad \varepsilon = \frac{E}{e\Lambda_i N_n},$$

where N_n is the density of neutral atoms.

Eliminating ε from (2.1) and (2.2), we find with the help of (2.3) that

$$4\pi n(n_+ - n) + \frac{a_0^{(i)}}{\xi_i^{i-1}} \frac{d}{d\xi_i}\left(\xi_i^{i-1} \frac{dn}{d\xi_i}\right) + \frac{4\pi}{\xi_i^{i-1}} \int\limits_0^{\xi_i} t_i^{i-1}(n_+ - n)\, dt_i \frac{dn}{d\xi_i} - a_1 n n_+ + a_2 n = 0\,, \tag{2.4}$$

$$4\pi n_+(n_+ - n) - \frac{\mu a_0^{(i)}}{\lambda \xi_i^{i-1}} \frac{d}{d\xi_i}\left(\xi_i^{i-1} \frac{dn_+}{d\xi_i}\right) + \frac{4\pi}{\xi_i^{i-1}} \int\limits_0^{\xi_i} t_i^{i-1}(n_+ - n)\, dt_i \frac{dn_\pm}{d\xi_i} + \mu a_1 n n_+ - \mu a_2 n = 0\,. \tag{2.5}$$

We seek a solution of the nonlinear system (2.4), (2.5) in the form

$$n = n_{0i} G_i(\xi_i) + (\xi_{i\Sigma}^2 - \xi_i^2) \sum_{k=1} p_k \xi_i^{2k+2}\,, \tag{2.6}$$

$$n_+ = n_{0+i} G_i(\xi_i) + (\xi_{i\Sigma}^2 - \xi_i^2) \sum_{k=1} q_k \xi_i^{2k+2}\,; \tag{2.7}$$

Here, n_{0i} and n_{0+i} are the dimensionless electron and ion densities at the margin, $\xi_{i\Sigma} = x_{i\Sigma}/\Lambda_i$ is the value of ξ_i at the boundary of the region, and $G_i(\xi_i)$ satisfies Eq. (1.6), yielding Eqs. (1.7)–(1.9) for i = 1, 2, 3.

It is easy to see that the choice of substitution (2.6), (2.7) satisfies the boundary conditions of the problem. We substitute (2.6) and (2.7) into (2.4) and (2.5), expanding G_i in series form:

$$G_i = 1 - \frac{\xi_i^2}{2i} + \sum \alpha_k^{(i)} \xi_i^{2k}\,,$$

where $\alpha_k^{(i)}$ are known coefficients corresponding at i = 1 to the power-series expansion of the cosine and corresponding at i = 2 to the expansion of the zero-order Bessel function, etc.

Comparing coefficients of identical powers of ξ_i, we find, in particular, for the zeroth-degree coefficients of ξ_i the following:

$$4\pi n_{0i}(n_{0+i} - n_{0i}) - a_0^{(i)} n_{0i} - a_1 n_{0i} n_{0+i} + a_2 n_{0i} = 0\,, \tag{2.8}$$

$$4\pi n_{0+i}(n_{0+i} - n_{0i}) + \frac{\mu}{\lambda} a_0^{(i)} n_{0+i} + \mu a_1 n_{0i} n_{0+i} - \mu a_2 n_{0i} = 0\,. \tag{2.9}$$

In an analogous manner, we can obtain recurrence relations for the coefficients p_k and q_k and (2.6), (2.7). In particular, for i = 1 (point symmetry), we have (omitting the subscript for simplicity)

$$\Bigg\{\frac{(-1)^k[4\pi n_0(n_{0+} - n_0)(2^{2k} - 1) - a_1 n_0 n_{0+}(2^{2k-1} - 1)]}{(2k)!} + A_k + (1+2k)B_k - \frac{A_{k-1}}{2k-1} + \frac{1}{4\pi}(a_2 - a_1 n_{0+})C_k + \frac{a_0(2k+2)(2k+1)}{4\pi} C_{k+1} - a_1 n_0 D_k + \sum_{j=1}^{k-1}\Bigg[\left(\frac{(-1)^j}{(2j)!} + \frac{C_j}{4\pi n_0}\right) A_{k-j-1} + \frac{(-1)^j}{(2j)!}\left(1 + \frac{2(k-j-1)}{2j+1}\right) B_{k-j-1} - \frac{(-1)^j}{(2j+1)!} \frac{A_{k-j-2}}{2(k-j)-3} + \frac{2(k-j)A_{j-1}}{4\pi n_0(2j-1)} C_{k-j} - \frac{(-1)^j n_{0+} a_1 C_{k-j-1}}{4\pi(2j)!} - \left(\frac{a_1 C_j}{4\pi} + \frac{(-1)^j}{(2j)!} a_1 n_0\right) D_{k-j-1}\Bigg]\Bigg\} = 0\,, \tag{2.10}$$

$$\Bigg\{\frac{(-1)^k[4\pi n_{0+}(n_{0+} - n_0)(2^{2k} - 1) + \mu a_1 n_0 n_{0+}(2^{2k-1} - 1)]}{(2k)!} + A_k^+ + (1+2k)B_k^+ - \frac{A_{k-1}^+}{2k-1} + \frac{\mu a_1 n_0 C_k^+}{4\pi} - \frac{\mu a_0(2k+2)(2k+1)}{4\pi\lambda} C_{k+1}^+ + (\mu a_1 n_{0+} - \mu a_2) D_k^+ + \sum_{j=1}^{k-1}\Bigg[\left(\frac{(-1)^j}{(2j)!} + C_j^+\right) A_{k-j}^+ + \frac{(-1)^j}{(2j)!}\left(1 + \frac{2(k-j-1)}{2j+1}\right) B_{k-j-1}^+ + \frac{(-1)^k}{(2k+1)!} \frac{A_{k-j-2}}{2(k-j)-3} + \frac{A_{j-1}^+ 2(k-j) C_{k-j}^+}{(2j-1)4\pi n_{0+}} + \left(\frac{\mu a_1 n_0 (-1)^j}{(2j)!\,4\pi} + \mu a_1 D_j^+\right) C_{k-j-1}^+ + \frac{(-1)^j}{(2j)!} \mu a_1 n_{0+} D_{k-j-1}^+\Bigg]\Bigg\} = 0\,. \tag{2.11}$$

Here,

$$A_k = \pi^3 n_0(q_{k-1} - p_{k-1}) - 4\pi n_{0+}(q_{k-2} - p_{k-2})\,,$$

$$B_k = \pi^3(n_{0+} - n_0)p_{k-1} - 4\pi(n_{0+} - n_0)p_{k-2}\,,$$

$$C_k = \pi^3 p_{k-1} - 4\pi p_{k-2}, \quad D_k = {}^1\!/_4\pi^2 q_{k-1} - q_{k-2}, \quad A_k = \pi^3 n_{0+}(q_{k-1} - p_{k-1}) - 4\pi n_{0+}(q_{k-2} - p_{k-2}), \quad B_k^+ = \pi^3(n_{0+} - n_0)q_{k-1} - 4\pi(n_{0+} - n_0)q_{k-2}\,,$$

$$C_k^{+} = \pi^3 q_{k-1} - 4\pi q_{k-2},\ D_k^{+} = {}^1/_4\pi^2 p_{k-1} - p_{k-2}\ .$$

For example, we have

$$p_1 = \frac{12\pi n_0 (n_{0+} - n_0) - a_1 n_0 n_{0+}}{3!\, a_0 \pi^2}, \quad q_1 = -\frac{12\pi n_{0+} (n_{0+} - n_0) + \mu a_1 n_0 n_{0+}}{3! a_0 \mu \pi^2},$$
$$p^2 = \left[\frac{4}{\pi^2} - \frac{4\pi (n_{0+} - n_0)}{3!\, a_0} - \frac{a_2}{5\cdot 6 a_0} - \frac{1}{6}\right] p_1 + \frac{a_1 n_0 n_{0+}}{3\cdot 5\cdot 6 a_0 \pi^2} - \frac{4\pi a_0 (q_1 - p_1)}{5\cdot 6 a_0},$$
$$q_2 = \left[\frac{4\lambda}{\pi^2} + \frac{4\pi\lambda (n_{0+} - n_0)}{3!\, \mu a_0} + \frac{4\lambda a_1 n_0 \pi}{3!\, a_0} + \frac{\lambda}{6}\right] + \frac{4\lambda^2 a_2 p_1}{(5\cdot 6)^2 \mu^2 a_0 \pi^2} + \frac{16\lambda^2 n_0 (q_1 - p_1)}{(5\cdot 6)^2 \mu^2 a_0} + \frac{\lambda a_1 n_0 n_{0+}}{3!\, 5\cdot 6 a_0 \pi^2}\ .$$

It is easy to show that under free-diffusion conditions, for which recombination and the effect of the space-charge field can be neglected, the coefficients $p_k^{(i)}$ and $q_k^{(i)}$ tend toward zero, and the electron and ion density distributions are given by the function

$$G_i(\xi_i) \quad (i = 1, 2, 3)\ .$$

Let us now demonstrate the convergence of the series in (2.6), (2.7) for the case i = 1 (plane symmetry). Analyzing recurrence relations (2.10) and (2.11), we can easily show that the inequality $p_k < 4\pi^{-2} p_{k-1}$ becomes valid at some $k \geq k_0$; it follows that $p_k < (4\pi^{-2})^{n-1} p_\alpha$ where p_α is some finite number. Accordingly, series (2.6) for $k \geq k_0$ is majorized by a power series with the common term $(4\pi^{-2})^n x^{2n}$, having a convergence radius of $\pi/2$.

Series (2.6) thus converges absolutely in this region. The convergence of series (2.7) can be proved in an anologous manner, as can the convergence of series (2.6) and (2.7) for i = 2, 3.

3. System (2.8), (2.9) for n_{0+i}, n_{0i} is identical for all i (i = 1, 2, 3); i. e., it retains its form in different geometric problems. The geometry is taken into account by the quantity Λ_i in $a_0^{(i)}$. Accordingly, the results obtained below from an analysis of (2.8), (2.9) will be valid for any problem geometry.

Converting in (2.8), (2.9) from dimensionless to dimensional variables and parameters, and solving the system for N(0) and $N_+(0)$ under the conditions

$$\mu = \frac{b}{b_+} \gg 1, \quad \alpha \ll \alpha_0 = 4\pi e b\ , \tag{3.1}$$

which are obviously always satisfied, we find

$$1 - \frac{N_+(0)}{Z}\alpha - \frac{D_a}{Z\Lambda^2_i} - \frac{\alpha}{\alpha_0} \quad \left[1 - \frac{D}{Z\Lambda_i^2}\right]\left[1 - \frac{Z}{N_+(0)\alpha}\right] = 0\ , \tag{3.2}$$
$$\frac{N(0)}{N_+(0)} - \left[N_+(0) + \frac{\mu}{\lambda}\,\frac{D}{\alpha_0 \Lambda_i^2}\right]\left[N_+(0) + \mu\frac{Z}{\alpha_0}\right]^{-1} \quad \left(D_a = \frac{b_+ D + D_+ b}{b_+ + b}\right), \tag{3.3}$$

where D_a is the ambipolar diffusion coefficient. Equation (3.2) is the steady-state condition; it is equivalent to the condition $\partial/\partial t = 0$ which permits us to calculate the density in the central region from the diffusion length Λ_i for given parameters z, α, D, D_+, b, b_+ and for a given geometry of the plasma region. This geometry is reflected in the steady-state condition. For certain ratios between the parameters in (3.2) and (3.3), particular forms of the steady-state conditions can be obtained for the familiar "pure regimes".

For free diffusion, as an example, we have

$$Z = \frac{D}{\Lambda_i^2}, \quad \frac{N(0)}{N_+(0)} = \frac{D_+}{D} \quad \text{for} \quad N_+(0) \ll \frac{D}{\alpha_0 \Lambda_i^2}, \quad \frac{Z}{\alpha_0};$$

for ambipolar diffusion we have

$$Z = \frac{D_a}{\Lambda_i^2}, \quad N(0) \approx N_+(0) \quad \text{for} \quad \frac{D}{\alpha_0 \Lambda_i^2}, \quad \frac{Z}{\alpha_0} \ll N(0) \ll \frac{Z}{\alpha_0}\ .$$

For the recombination regime, we have

$$N(0) = N_+(0) = \frac{Z}{\alpha} \quad \text{for} \quad N \gg \frac{D}{\alpha_0 \Lambda_i^2}, \quad \frac{Z}{\alpha_0}.$$

For the transition regimes from free diffusion to ambipolar diffusion and from ambipolar to recombination, Eqs. (3.2) and (3.3) must be used.

Significantly, the steady-state conditions unambiguously relate only the density at the origin with the parameters of the problem; they are not directly connected to the form of the density distribution function, which can be calculated from the recurrence relations. We turn now to conditions (3.1). As is easy to see from evaluations, these conditions always hold for a plasma consisting of electrons and ions. According to evaluations based on known recombination coefficients, conditions (3.1) must be satisfied for almost all known cases. However, if the opposite condition,

$$\alpha \gg \alpha_0 = 4\pi eb, \tag{3.4}$$

is held possible which, generally speaking, may be true in the case of induced photorecombination in an intense radiation field at high neutral-gas pressures, we can find a steady-state condition analogous to (3.4) from the equations for n_+ and n in this case; i. e., the steady state must always be recombinational, regardless of the maximum electron density.

4. We turn now to the analogous steady-state problem for the case in which the ionization is localized in a certain effective volume and in which the boundaries are essentially at infinity. Localization of the ion can be achieved by introducing a certain distribution parameter in Eqs. (1.1)–(1.3). For the spherically symmetric problem, we introduce this factor by replacing Z by $Z \exp-(r/R_0)^2$, where R_0 is the characteristic dimension of the region in which the ionization is localized. After calculations analogous to those in section 2, we find for this case a system of equations similar to system (2.4), (2.5) for the bounded problem:

$$4\pi n (n_+ - n) + \frac{C_0}{\eta^2} \frac{d}{d\eta} \left(\eta^2 \frac{dn}{d\eta} \right) + \frac{4\pi}{\eta^2} \int_0^{\eta} t^2 (n_+ - n)\, dt \, \frac{dn}{d\eta} - C_1 n n_+ + C_2 n \exp -(\eta^2) = 0, \tag{4.1}$$

$$4\pi n_+ (n_+ - n) - \frac{\mu C_0}{\lambda \eta^2} \frac{d}{d\eta} \left(\eta^2 \frac{dn_+}{d\eta} \right) + \frac{4\pi}{\eta^2} \int_0^{\eta} t^2 (n_+ - n) dt \, \frac{dn_+}{d\eta} + C_1 \mu n n_+ - \mu C_2 n \exp(-\eta^2) = 0, \tag{4.2}$$

where

$$\eta = \frac{r}{R_0}, \quad C_0 = \frac{D}{ebN_0 R_0^2}, \quad C_1 = \frac{\alpha}{eb}, \quad C_2 = \frac{Z}{beN_n}.$$

We seek a solution of (4.1), (4.2) satisfying the boundary conditions

$$n_+ /_{\eta \to \infty} = n /_{\eta \to \infty} = 0$$

and the condition of boundedness at the origin, in the form

$$n = e^{-\eta^2} (n_0 + \sum_{k=0} p_k \mu^{2k+4}), \quad n_+ = e^{-\eta^2} (n_{0+} + \sum_{k=0} q_k \eta^{2k+4}). \tag{4.3}$$

Here n_0, n_{0+} are the dimensionless electron and ion densities at the origin. Substitution of (4.3) into (4.1) and (4.2), with a power-series expansion of $\exp(-\eta^2)$, yields equations analogous to (2.8) and (2.9) for the bounded problem for the density at the origin:

$$4\pi n_0 (n_{0+} - n_0) - C_0 n_0 - C_1 n_0 n_{0+} + C_2 n_0 = 0, \tag{4.4}$$

$$4\pi n_{0+} (n_{0+} - n_0) + \mu/\lambda C_0 n_{0+} + \mu C_1 n_0 n_{0+} - \mu C_2 n_0 = 0. \tag{4.5}$$

Recurrence relations for the coefficients p_k and q_k in (4.3) can be obtained in a completely analogous manner. Because of the similarity of (4.4), (4.5) and (2.8), (2.9), and when conditions in (3.1) hold, we can obtain from (4.4) and (4.5) steady-state conditions analogous to (3.2) and (3.3); a difference is that here the role of the diffusion length is played by the quantity

$$\Lambda_\infty = {}^1/_6 R. \tag{4.6}$$

The numerical coefficient in (4.6) is directly related to the approximation chosen for the ionization-frequency distribution.

All the results obtained in the study of steady-state regimes for the bounded balance problem naturally remain valid for this case, with the replacement of Λ_i by Λ_∞.

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